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TECHNICAL NOTES

EFFECTS OF THE THERMAL GRADIENT ON THE MOISTURE* MIGRATION IN A CYLINDRICAL CONCRETE VESSEL

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BY ASSUMING the existence of local thermal equilibrium in the porous system, and the gaseous vapor phase as the major factor of mass transfer, one can derive the basic equations for heat and mass transfer from the laws of conservation of mass, momentum, and energy, and the kinetic theory of ideal gases. For mass and heat transfer in a porous concrete, those can be given in the following form (the Nomenclature and the derivation of these equations are given in ref. [1]):

$$\begin{aligned} A_i\dot{\phi}+B_i\dot{P}+C_i\dot{T} = {} & D_i\nabla^2\phi+E_i\nabla^2P+F_i\nabla^2T \\ & +G_i(\nabla\phi)^2+H_i(\nabla P)^2 \\ & +I_i(\nabla T)^2+J_i(\nabla\phi\cdot\nabla P) \\ & +K_i(\nabla\phi\cdot\nabla T)+L_i(\nabla P\cdot\nabla T), \end{aligned} \qquad (i=1,2,3), \quad (1)$$

where the coefficients $A_i,\dots,L_i$ are functions of the dependent variables ϕ, P, T and ε_g. They are defined as follows:

$$\begin{aligned} A_1 &= \bar{Y}(\partial\varepsilon_g/\partial\phi)+\varepsilon_g & A_2 &= (1-\phi)(\partial\varepsilon_g/\partial\phi)-\varepsilon_g \\ B_1 &= \bar{Y}(\partial\varepsilon_g/\partial P)+\varepsilon_g(\phi/P) & B_2 &= (1-\phi)\bar{O}_p \\ C_1 &= \bar{Y}(\partial\varepsilon_g/\partial T)-\varepsilon_g(\phi/T) & C_2 &= -(1-\phi)\bar{O}_T \\ D_1 &= D\varepsilon_g(M_a/M) & D_2 &= -D\varepsilon_g(M_w/M) \\ E_1 &= \zeta\phi(k_g^0/\eta_g) & E_2 &= \zeta(1-\phi)(k_g^0/\eta_g) \\ F_1 &= 0 & F_2 &= 0 \\ G_1 &= -D\bar{O}_\phi(M_a/M) & G_2 &= D\bar{O}_\phi(M_w/M) \\ H_1 &= \phi\bar{W}_p(k_g^0/\eta_g) & H_2 &= (1-\phi)\bar{W}_p(k_g^0/\eta_g) \\ I_1 &= 0 & I_2 &= 0 \\ J_1 &= D\bar{O}_p(M_a/M) & J_2 &= -D\bar{O}_p(M_w/M) \\ &\quad +(k_g^0/\eta_g)[\phi(\partial\zeta/\partial\varepsilon_g) & &\quad +(k_g^0/\eta_g)[(1-\phi)(\partial\zeta/\partial\varepsilon_g) \\ &\quad \times(\partial\varepsilon_g/\partial\phi)+\zeta] & &\quad \times(\partial\varepsilon_g/\partial\phi)-\zeta] \\ K_1 &= -D\bar{O}_T(M_a/M) & K_2 &= D\bar{O}_T(M_w/M) \\ L_1 &= -\phi\bar{W}_T(k_g^0/\eta_g) & L_2 &= -(1-\phi)\bar{W}_T(k_g^0/\eta_g) \end{aligned}$$

$$\begin{aligned} A_3 &= \rho_w Q(\partial\varepsilon_g/\partial\phi) \\ B_3 &= \rho_w Q(\partial\varepsilon_g/\partial P)-\varepsilon_g \\ C_3 &= \rho_w Q(\partial\varepsilon_g/\partial T)+[(\varepsilon-\varepsilon_g)\rho_w(C_p)_w+\rho_s(1-\varepsilon)(C_p)_s] \\ &\quad +\varepsilon_g P[\phi M_w(C_p)_v+(1-\phi)M_a(C_p)_s]/RT \\ D_3 &= 0, \quad E_3 = 0, \quad G_3 = 0, \quad H_3 = 0 \\ F_3 &= K \\ I_3 &= (\partial K/\partial\varepsilon_g)(\partial\varepsilon_g/\partial T) \\ K_3 &= (\partial K/\partial\varepsilon_g)(\partial\varepsilon_g/\partial\phi)-DM_aM_w\varepsilon_g P[(C_p)_a-(C_p)_v]/RTM \\ L_3 &= (\partial K/\partial\varepsilon_g)(\partial\varepsilon_g/\partial P)+(K_g^0\zeta P)[M_w(C_p)_v\phi \\ &\quad +M_a(C_p)_a(1-\phi)]/RT \end{aligned}$$

in which

$$\begin{aligned} \bar{O}_\phi &= \varepsilon_g[(M_w-M_a)/M]-(\partial\varepsilon_g/\partial\phi) \\ \bar{O}_p &= (\varepsilon_g/P)+(\partial\varepsilon_g/\partial P) \\ \bar{O}_T &= (\varepsilon_g/T)+(\partial\varepsilon_g/\partial T) \\ \bar{Y} &= \phi-(\rho_w RT/M_w P) \\ \bar{W}_p &= (\zeta/P)+(\partial\zeta/\partial\varepsilon_g)(\partial\varepsilon_g/\partial P) \\ \bar{W}_T &= (\zeta/T)-(\partial\zeta/\partial\varepsilon_g)(\partial\varepsilon_g/\partial T). \end{aligned}$$

For an axisymmetric boundary-value problem, the operators shown in equation (1) are expressed as

$$\nabla^2 = \frac{1}{R_1}\frac{\partial}{\partial R_1}\left(R_1\frac{\partial}{\partial R_1}\right) \quad \text{and} \quad \nabla = \frac{\partial}{\partial R_1}.$$

By examining the three basic equations (1), we note that there are four variables $\phi(R_1,t)$, $P(R_1,t)$, $T(R_1,t)$ and $\varepsilon_g(R_1,t)$. Therefore, equations (1) should be incorporated with a constitutive equation, known as the liquid-vapor equilibrium curve (Fig. 1), to form a set of governing differential equations.

The boundary conditions on the inner surface of the concrete cylinder, on which the moisture barrier is installed, are

$$R_1 = R_i\text{:}\ \frac{\partial\phi}{\partial R_1} = 0, \quad \frac{\partial T}{\partial R_1} = -\frac{q}{K}, \quad \text{and} \quad \frac{\partial P}{\partial R_1} = 0. \qquad (2)$$

On the outer surface of the concrete cylinder, the moisture is removed by natural causes. The boundary conditions are

$$\begin{aligned} R_1 = R_0\text{:}\ \frac{\partial\phi}{\partial R_1} &= -\alpha\frac{RT}{PD}(\phi-\phi_{01}), \\ \frac{\partial T'}{\partial R_1} &= -\frac{h}{k}(T-T_{01}), \\ P &= P_{01}. \end{aligned} \qquad (3)$$

The initial conditions for the present study are given by the following constants:

$$\phi(R_1,0) = \phi_i, \quad P(R_1,0) = P_i, \quad \text{and} \quad T(R_1,0) = T_i. \quad (4)$$

Therefore, equations (1)-(4) and the constitutive equation form a well-posed nonlinear boundary-value problem for the simultaneous mass and heat transfer in the concrete cylinder shown in Fig. 2.

An implicit finite-difference scheme is employed to obtain numerical results. The first step is to replace the equations by a

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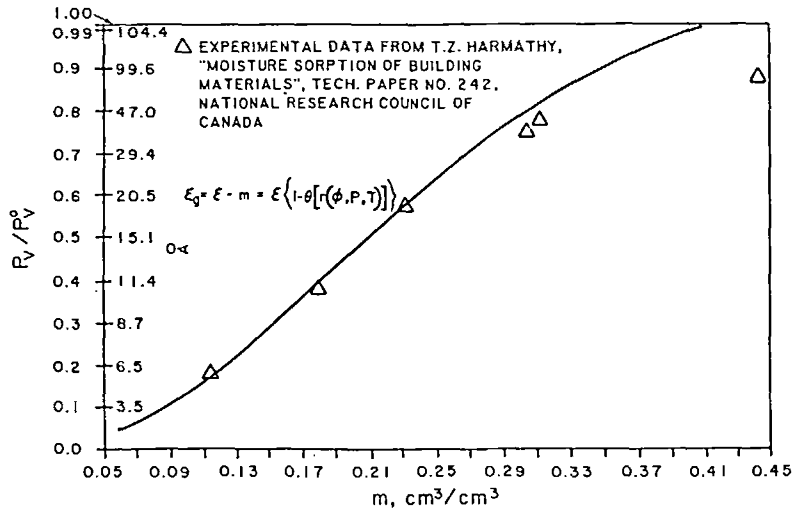

FIG. 1. Liquid–vapor equilibrium curve for a lightweight concrete.

set of algebraic finite backward-in-time equations, which give the relationships among the dependent variables ϕ, P, and T at neighboring points in an (R_1, t) space. The numerical solution of the simultaneous algebraic equations thus obtained yields the values of the dependent variables at the pre-assigned grid points throughout the domain investigated. In order to achieve a stable computation, a value of Δt must be used at the initial stage of the computational simulation which is smaller than that at the advanced stages, due to errors associated with the initial guess of sorption equilibrium.

Figures 3–5 show the moisture distributions in a porous concrete cylindrical tube with different intensities of heat flux ($q = 0.5 \times 10^2$, 0.8×10^2, 1.1×10^2 kg s^{-3}) imposed on the inner wall surface. At a stage of high pore saturation or the funicular saturation stage, a considerable amount of moisture loss has occurred in both the hot and the cold zones. The

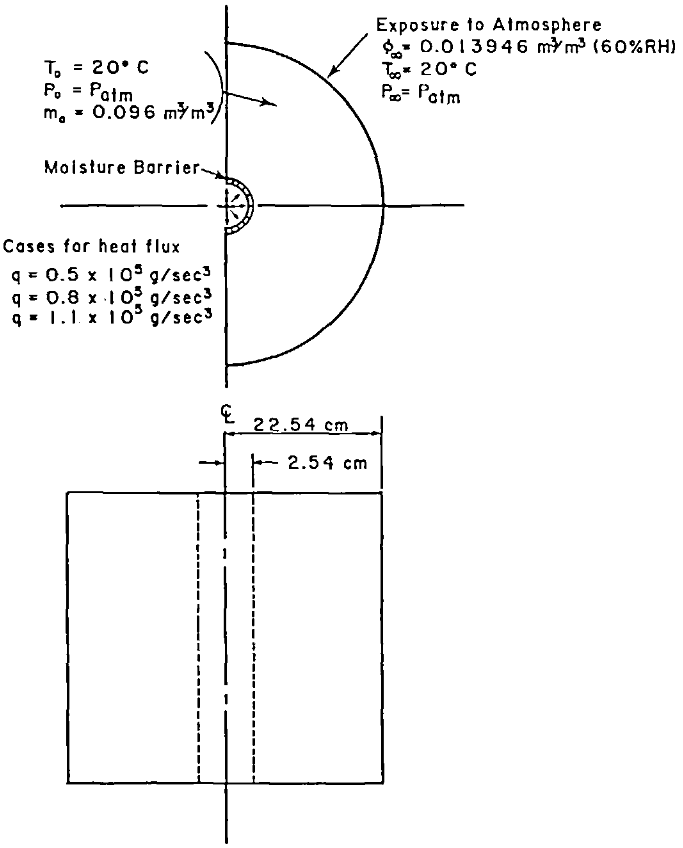

FIG. 2. Porous concrete vessel.

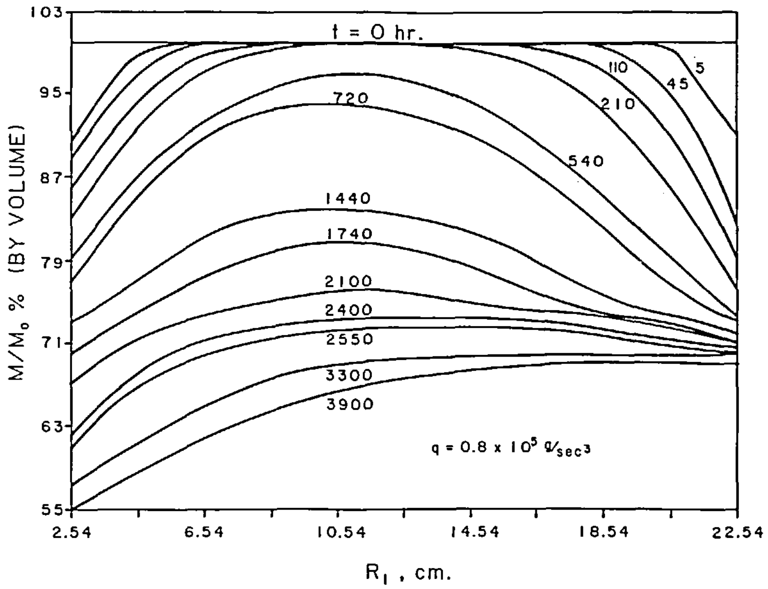

FIG. 3. Moisture distribution in radial direction.

intensity of heat flux affects the rate of moisture losses locally. The higher intensity of heat flux imposed on the inner surface of the cylinder results in a higher drying rate in the heated surface zone only. A steep temperature gradient provides the large thermal driving force to transport the moisture in the hot inner zone outward into the middle zone. On the outer unheated surface exposed to the natural ambient surroundings, the moisture loss, caused by the mass diffusion, is rather high. The drying rate of moisture is relatively independent of the properties of the porous concrete and the properties of fluid moisture; it depends strongly on the parameters characterizing the surroundings of the system. In the middle zone between two surfaces, only a meagre amount of moisture loss has been observed. Both diffusion and thermal driving force are inactive in this region. These phenomena have been reported in refs. [2, 3]. When the drying proceeds to a stage called pendular stage, capillary action and evaporation–condensation occur in the pore spaces. Both the internal and the topological characteristics of porous concrete and the intensity of a heat flux play the dominant role in the migration of moisture. Moisture distribution curves for this stage, shown in Figs. 3–5, are no longer convex, while the moisture distribution curves resulting from the molecular diffusion alone are convex. In addition, the rate of moisture loss is faster near the heated surface than that at the outer exposed surface. The moisture content in the hot zone has a tendency to travel outward in the negative direction of temperature gradient. Obviously, an increase in heat flux generally increases the mobility of the water molecules, the viscosity of water, and the adhesive forces existing between the solid and the absorbed

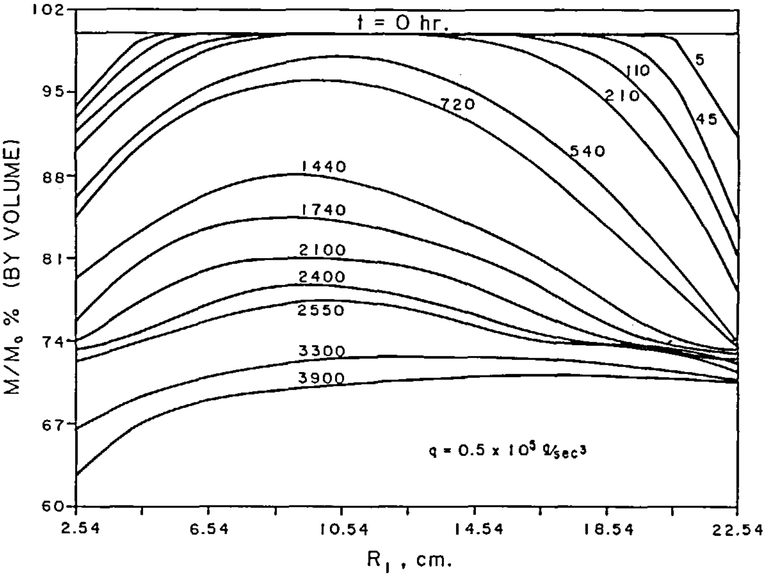

FIG. 4. Moisture distribution in radial direction.

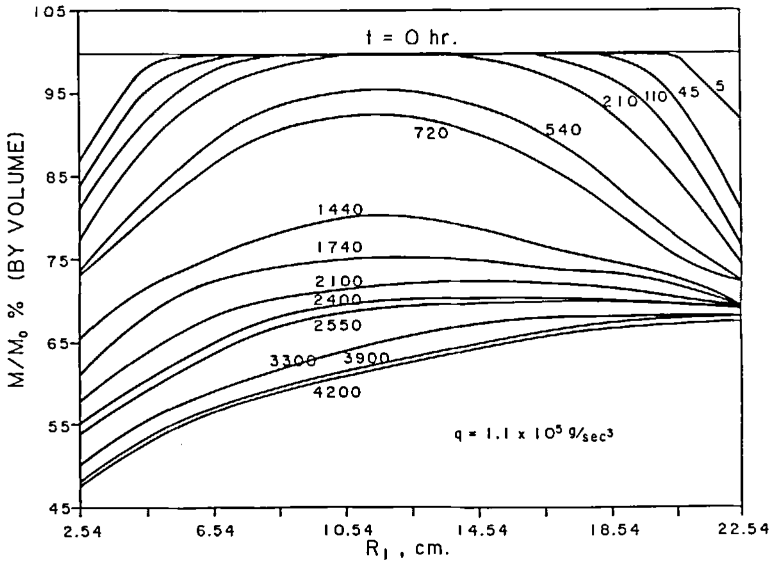

FIG. 5. Moisture distribution in radial direction.

liquid layers. This phenomenon results in accelerating the removal of gel and capillary water in concrete. Therefore, the drying rate and the moisture loss at any time are functions of temperature; the drying rate increases rapidly with increase of temperature. Similar results have been reported previously [4].

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DIFFUSION AND INFORMATION

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NOMENCLATURE

a_1, a_2	normalizing constants
c	concentration
c_0	initial concentration
c_e	final concentration
D	diffusivity
FI	functional information
H	differential entropy
L	length of diffusion cell
p	density distribution function
t	time
x	distance

AS A MEASURE of the disorder of a continuous distribution with the density distribution function $p(x)$, the differential entropy can be calculated in the following way [1]:

$$H = -a_1 \int_{-\infty}^{\infty} p(x) \log p(x)\, dx, \qquad (1)$$

where a_1 is a constant.

In distributions with a large number of particles, e.g. as is usually present in diffusion processes, the density distribution of the residence probability $p(x)$ in equation (1) can be replaced by that of an appropriately normalized concentration $c(x)$.

On the other hand, for time-dependent concentration